Computer Science 385
Design and Analysis of Algorithms
Siena University
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Problem Reductions Practice

Problem Reduction

Idea: solving a problem by using an existing solution to another problem.

An example from programming:

You wish to draw a circle, and already have a method to draw an ellipse:

```
draw_ellipse(double horiz, double vert, double x, double y)
```

So we use it in our implementation of

```
draw_circle(double radius, double x, double y) {






}
```

We have *transformed* or *reduced* the problem of drawing a circle to the problem of drawing an ellipse.

We have "proven" that drawing a circle is ________________________________ drawing an ellipse.

Example 1: The Pairing Problem

Problem statement: given two n-element arrays $A1$ and $A2$. Your task is to rearrange the values in $A2$ such that the smallest value in $A2$ is paired with the smallest value in $A1$. The second smallest in $A2$ is paired with the second smallest in $A1$, and so one. Only values of $A2$ are rearranged; $A1$ is unchanged.

Example Input:

$A1$	23	5	57	45
$A2$	150	175	100	120

Output:

$A1$				
$A2$				

Assume we have some algorithm that can solve this.

Now consider the sorting problem for an n-element array, $A[0..n-1]$. How can we use the solution to the pairing problem to solve the sorting problem?

What is the total cost of this approach?

We have *reduced* the sorting problem to an instance of the pairing problem.

Using problem reduction to show a lower bound

- If problem A is **at least as hard as** problem B, then a lower bound for B is also a lower bound for A.
- Hence, we wish to find a problem B with a known lower bound that can be reduced to the problem A.

In our example, problem A is the ____________________ problem

problem B is the ____________________ problem

Problem B (the ____________________ problem) has a known lower bound of $\Omega(\qquad)$.

We reduced Problem B (the ____________________ problem) to an instance of problem A (the ____________________ problem)

Therefore, problem A (the ____________________ problem) is at least as hard as problem B (the ____________________ problem)

So problem A (the ____________________ problem) also has a lower bound of $\Omega(\qquad)$

What about Big-Θ?

Example 2: Euclidean Minimum Spanning Tree (MST)

Problem statement: given n points in the plane, construct a tree of minimum total length whose vertices are the given points.

A problem with a known lower bound to use to establish a lower bound for Euclidean MST:

Goal: reduce an instance of the ____________________ problem

to an instance of the ____________________ problem

Element uniqueness problem input: a set of numbers $x_1, x_2, \cdots, x_n$

Element uniqueness problem known lower bound: $\Omega(n \log n)$, tight

Steps:

1. Transform the element uniqueness inputs into a set of inputs for Euclidean MST (which is a set of points in the plane).

2. Solve Euclidean MST on that input
3. Use this solution to get a solution to element uniqueness

What did we show?