Computer Science 385
Design and Analysis of Algorithms
Siena College
Spring 2025

Decrease and Conquer Practice

ALGORITHM InsertionSort(A)
  //Input: an array $A[0..n-1]$
  for $i \leftarrow 0..n-1$ **do**
    $v \leftarrow A[i]$
    $j \leftarrow i-1$
    while $j \geq 0$ **and** $A[j] < v$ **do**
      $A[j+1] \leftarrow A[j]$
      $j \leftarrow j-1$
    $A[j+1] \leftarrow v$

In-place?

Stable?

Basic operations:

Best/average/worst cases?

Is this best case important? In what use cases is this more likely?

Number of comparisions, worst case:

$C_{worst}(n) =$

Number of comparisions, best case:

$C_{best}(n) =$

Number of comparisons, average case (on random ordered data):

$C_{avg}(n) \approx$

ALGORITHM FACTORIAL(n)
  if $n = 0$ **then**
    return 1
  else
    return $n \cdot$ FACTORIAL$(n - 1)$

The size is n (the parameter passed in to get things started) and the basic operation is the multiplication in the `else` part.

There is no difference among the best, average, and worst cases.

Recurrence:

$M(n) =$

Base case/initial condition:

$M(\quad) =$

Back substitution steps:

Pattern:

$M(n) =$

Application of base case, and result:

$M(n) =$

Towers of Hanoi

Recall that solving an instance of this problem for n disks involves solving an instance of the problem of size $n-1$, moving a single disk, then again solving an instance of the problem of size $n-1$. We denote the number of moves to solve the probem for n disks as $M(n)$.

Recurrence with base case:

$M(n) =$

$M(\quad) =$

Backward substitution:

$M(n) =$

Pattern:

$M(n) =$

Application of base case and result:

$M(n) =$

ALGORITHM BinDigits(n)
  if $n = 1$ **then**
    return 1
  else
    return BinDigits($\lfloor \frac{n}{2} \rfloor$) $+ 1$

In this case, we will count the number of additions, $A(n)$.

Recurrence with base case:

$A(n) =$

$A(\quad) =$

Convert to a power of 2 (smoothness rule: let $n = 2^k$):

$A(2^k) =$

$A(2\quad) =$

Backward substitution:

$A(2^k) =$

Let $i =$, to sub into this pattern:

$A(2^k) =$

Application of base case and result, convert back to n:

$A(2^k) =$

$A(n) =$

Another example of a common pattern

$$C(n) = 2C(n/2) + 2$$

when $n > 0$, and a base case of $C(1) = 0$.

ALGORITHM InsertionSort(A)
//Input: an array $A[0..n-1]$
$recInsertionSort(A, n-1)$

ALGORITHM RecInsertionSort(A, max)
//Input: an array $A[0..n-1]$
//Input: upper index limit to sort max
// Base case: a 1-element array
if $max = 0$ **then**
  return
// Recursive case: sort first $max - 1$
$RecInsertionSort(A, max - 1)$
// now insert max'th in correct location
$v \leftarrow A[max]$
$j \leftarrow max - 1$
while $j \geq 0$ **and** $A[j] > v$ **do**
  $A[j+1] \leftarrow A[j]$
  $j \leftarrow j - 1$
$A[j+1] \leftarrow v$

Worst case recursive analysis for the number of comparisons: