SIENA*college*
Computer Science

Computer Science 385
Analysis of Algorithms
Siena College
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Topic Notes: Iterative Improvement

Iterative improvement is another algorithm design technique for solving optimization problems. The general procedure involves:

- Starting with a feasible solution
- Repeat the following step until no improvement can be found: change the current feasible solution to a feasible solution with a better value of the objective function.
- Return the last feasible solution as optimal.

Typically, the change made at each step to the current solution is a small change and is obtained fairly quickly (just a local search).

The major difficulty is to deal with the problem of a *local optimum vs.*a *global optimum*. Local improvements may lead only to a local optimum.

The text lists several algorithms that proceed by iterative improvment. We will consider a couple.

Stable Marriage Problem

There is a set $Y = m_1, ..., m_n$ of n men and a set $X = w_1, ..., w_n$ of n women.

Each man has a ranking list of the women, and each woman has a ranking list of the men (with no ties in these lists).

A *marriage matching* M is a set of n pairs (m_i, w_j).

A pair (m, w) is said to be a *blocking pair* for matching M if man m and woman w are not matched in M but prefer each other to their mates in M.

A marriage matching M is called *stable* if there is no blocking pair for it; otherwise, its called *unstable*.

The *stable marriage problem* is to find a stable marriage matching for the men's and women's given preferences.

See example in Levitin Figrues 10.11 and 10.12 which demonstrates the *Gale-Shapley Algorithm*:

Step 0 Start with all the men and women being free.

Step 1 While there are free men, arbitrarily select one of them and do the following:

- Proposal: The selected free man m proposes to w, the next woman on his preference list
- Response: If w is free, she accepts the proposal to be matched with m. If she is not free, she compares m with her current mate. If she prefers m to him, she accepts m's proposal, making her former mate free; otherwise, she simply rejects m's proposal, leaving m free.

Step 2 Return the set of n matched pairs.

Notes:

- The algorithm terminates after no more than n^2 iterations with a stable marriage output.
- The stable matching produced by the algorithm is always *man-optimal*: each man gets the highest rank woman on his list under any stable marriage. One can obtain the *woman-optimal* matching by making women propose to men.
- A man (woman) optimal matching is unique for a given set of participant preferences.
- The stable marriage problem has practical applications such as matching med school graduates with hospitals for residency training.