SIENAcollege
Computer Science

Computer Science 210
Data Structures
Siena College
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Topic Notes: Trees

We've spent a lot of time looking at a variety of *linear* structures. There was a natural linear ordering of the elements in arrays, vectors, linked lists. We then put some restrictions on those structures, looking at stacks and queues and ordered linear structures.

Just like we can write programs that can branch into a number of directions, we can design structures that have branches.

Today, we'll start looking at our first more complicated structure: *trees*.

In a linear structure, every element has unique successor.

In a trees, an element may have many successors.

We usually draw trees upside-down in computer science.

You won't see trees in nature that grow with their roots at the top (but you can see some at Mass MoCA over in North Adams).

Examples of Trees

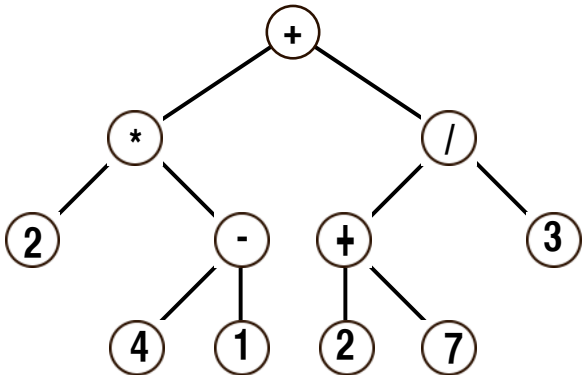
Expression trees

One example of a tree is an *expression tree*:

The expression

```
(2*(4-1))+((2+7)/3)
```

can be represented as

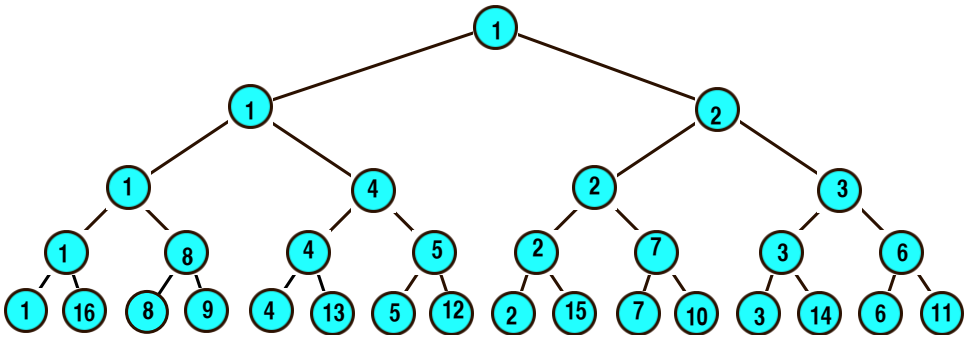


Once we have an expression tree, how can we evaluate it?

We evaluate left subtree, then evaluate right subtree, then perform the operation at root. The evaluation of subtrees is recursive.

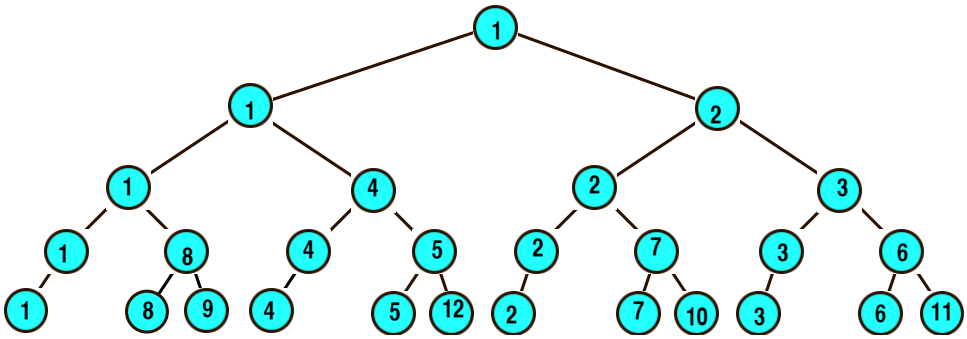
Tournament Brackets

Another example is a tree representing a tournament bracket:



(a *complete* and *full* tree)

or



(neither complete nor full)

Tree of Descendants

One popular use of a tree is a pedigree chart – looking at a person's ancestors. Instead, let's look at a person's descendants. (Example drawn in class).

Tree Definitions and Terminology

There are a lot of terms we will likely encounter when dealing with tree structures:

A tree is either *empty* or consists of a *node*, called the *root node*, together with a collection of (disjoint) trees, called its *subtrees*.

- An *edge* connects a node to its subtrees

- The roots of the subtrees of a node are said to be the *children* of the node.
- There may be many nodes without any successors: These are called *leaves* or *leaf nodes*. The others are called *interior nodes*.
- All nodes except root have unique predecessor, or *parent*.
- A collection of trees is called a *forest*.

Other terms are borrowed from the family tree analogy:

- *sibling* – nodes sharing the same parent
- *ancestor* – a node's parent, parent's parent, *etc.*
- *descendant* – a node's child, child's child, *etc.*

Some other terms we'll use:

- A *simple path* is series of distinct nodes such that there is an edge between each pair of successive nodes.
- The *path length* is the number of edges traversed in a path (equal to the number of nodes on the path - 1)
- The *height of a node* is length of the longest path from that node to a leaf.
- The *height of the tree* is the height of its root node.
- The *depth of a node* is the length of the path from the root to that node.
- The *degree of a node* is number of its direct descendents.
- The idea of the *level of a node* is defined recursively:
  - The root is at level 0.
  - The level of any other node is one greater than the level of its parent.

  Equivalently, the level of a node is the length of a path from the root to that node.

We often encounter *binary trees* – trees whose nodes are all have degree ≤ 2.

We will also orient the trees: each subtree of a node is defined as being either the *left* or *right*.

Binary Tree Interface

There are many possible specifications and implementations of binary trees that allow reasonable insertion and deletion of elements.

We will consider the one provided in the structure package first, and think about other possibilities later.

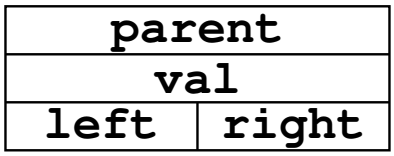
Unlike what we have seen for most of the structures to this point, the structure package does not define an interface for binary trees and then use implement that in one or more concrete classes. Structure simply implements the concrete class `BinaryTree`.

Unlike the linked list implementations, where we do not give the users of the structures access to the actual list nodes, the binary tree exposes more of its structure to users. The actual recursive data structure is given directly to users. Given this design decision, the implementation needs to ensure that changes that might be made by users cannot render a tree invalid. For example, if node A has a child reference to a node B, then it must always be the case the node B's parent reference is to A.

The tree is constructed of instances of class `BinaryTree`. Each `BinaryTree` object has the fields it needs to store its value and the parent and child references.

See Structure Source:
`structure5/BinaryTree.java`

A node has 4 fields, which we might draw as follows:



There are plenty of things to notice and think about here:

- We have three constructors.

    1. The first is used to create an "empty" `BinaryTree`. We will see this constructor used by the other constructors to create these empty trees in place of having `null` references to represent empty subtrees. This allows most methods to be called on these, eliminating lots of special cases. We could also use `null` to represent empty trees, but this would mean some extra code in several methods. Note that only empty tree nodes may contain a `null` value. Regular tree nodes must contain non-`null` values.
    2. The second constructor creates a tree node with no children (a leaf node) containing a particular value.
    3. The third constructor creates a tree node that may have children. Note that if a user of this constructor specifies a `null` child, it is replaces with an empty tree instance.

- We have accessors to retrieve the children or parent of a node.

- Note that the value and subtree links can be set by the user, but the parent reference is set only in a `protected` method. This is done to make sure we don't put the tree into a (bad) state mentioned above where a parent points to a child but the child doesn't point back to the parent.
- We have a variety of other (self-explanatory) methods to retrieve information about a tree: `size`, `root`, `height`, `depth`, `isFull`, `isEmpty`, `isComplete`. Note the recursive nature of many of these methods.

Binary Tree Example

We can construct a simple binary tree to represent and evaluate an arithmetic expression using the `BinaryTree` implementation:

```
((4+3)*(10-5))/2
```

See Example: BinaryExpressionTree

There are two versions of this program:

1. `BinaryExpressionTree.java` stores the operators and values to be used as `String`s.
   - This lets us use a `BinaryTree<String>`.
   - Since some are operators and some are numbers, we need to check and treat as appropriate, based on the contents of the `String`.
2. `BinaryExpressionTreeObject.java` stores the operators as `Character`s and the numbers as `Integer`s.
   - Here we instead use a `BinaryTree<Object>`, since that's the type that can represent both a `Character` and an `Integer`.
   - We check the actual type of the value retrieved with the `instanceof` operator and use the value as appropriate

Another option would be to define a common type along the lines of the `Token`s from the postscript lab.

In both cases, note the `treeString` method that prints our binary tree in a nice format.

Tree Traversals

Iterating over all values in our linear structures is usually fairly easy. Moreover, one or two orderings of the elements are the obvious choices for our iterations. Some structures, like an array or a

`Vector`, allow us to traverse from the start to the end or from the end back to the start very easily. A `SinglyLinkedList`, however, is most efficiently traversed only from the start to the end.

For trees, there is no single obvious ordering. Do we visit the root first, then go down through the subtrees to the leaves? Do we visit one or both subtrees before visiting the root?

We will consider 4 standard *tree traversals* for our binary trees:

1. *preorder*: visit the root, then visit the left subtree, then visit the right subtree.
2. *in-order* visit the left subtree, then visit the root, then visit the right subtree.
3. *postorder*: visit the left subtree, then visit the right subtree, then visit the root.
4. *level-order*: visit the node at level 0 (the root), then visit all nodes at level 1, then all nodes at level 2, etc.

For example, consider the preorder, in-order, and postorder traversals of the expression tree we looked at in the example code:

- preorder leads to prefix notation:
  / * + 4 3 - 10 5 2
- in-order leads to infix notation:
  4 + 3 * 10 - 5 / 2
- postorder leads to postfix notation:
  4 3 + 10 5 - * 2 /

The iterator concept fits nicely with tree traversals, but since the code for the iterators in the text is somewhat complex, so we will first consider traversals without iterators.

In our first traversal examples, we will build a small binary tree of `Integer` values and call methods that perform the traversal. Here, "visiting" a tree node involves passing its value to the method `process`.

See Example: BTTraversals

First, note the construction of the tree. We build the tree from bottom up, but do not store the subtrees in local variables during construction – we simply construct them in the parameters of the constructor for the next level up.

Now, consider each of the traversal implementations. The in-order, preorder, and postorder traversals work exactly as we would expect. Each is recursive, and we visit the subtrees and the root node as defined for each ordering.

The level-order traversal is a bit trickier. We need to visit the root of each subtree before doing anything in the next level. This calls for a queue!

For all of the others, we used a stack, just without thinking about it. We took advantage of the call stack to support the recursion!

Tree Iterators

The structure package has implementations of iterators for each of these four traversals. Whereas in the `do{pre,post,in}order` methods above, we were able to take advantage of the computer's run-time stack, we need to have a stack explicitly declared and used in the iterator implementations.

The complexity of the iterators varies with the type of traversal.

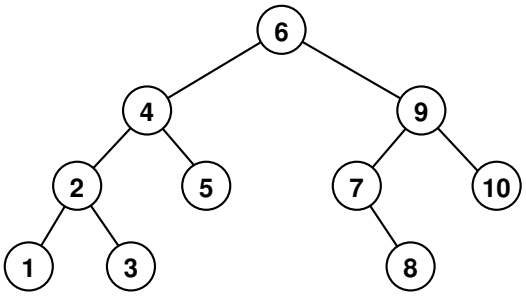
We need to make sure we get things onto the stack (or queue, in the case of the level order) in the right order.

At any time, we want the stack/queue to contain the tree nodes that still need to be visited.

Visiting a non-leaf node will result in additional nodes being added to the stack/queue.

In all cases, iteration can continue as long as something remains on the stack/queue.

We will consider this example tree:



In each case, recall that we need to satisfy the iterator interface (actually, the `AbstractIterator` in structure) by providing:

1. a constructor
2. a `reset` method
3. a `hasNext` method
4. a `next` method
5. a `get` method

We look at each in turn.

See Structure Source:
`structure5/BTPreorderIterator.java`

Our preorder traversal visits the root first, followed by the left subtree, then the right subtree.

Recall that we implicitly used the run-time stack for the non-iterator traversal code. Here, we manage the stack.

At any point, we want the tree node on the top of the stack to be the next tree node that needs to be visited.

We want to visit the root, then the left subtree, then the right subtree.

So to start or reset our iteration, we initialize the stack with the root node.

A `next` operation just involves popping, then processing the node on top of the stack, then pushing its right and left subtrees (in that order, since we want to process the left first).

Finally, we can implement `hasNext` by just checking if the stack is empty, which will tell is that the traversal has been completed.

See Structure Source:
`structure5/BTInorderIterator.java`

For this traversal, we need to visit the left subtree, then the root, then the right subtree.

Here, the first thing we want to visit is the deepest, leftmost child. So we need to initialize the state of our iterator so that that node (the deepest, leftmost child) is on top of the stack. To do this, we push the root, and all of the left subtrees until we come to a node which doesn't have a left subtree.

A `next` operation here involves popping the top value off the stack to be returned, then dealing with its right subtree. The first thing there that needs to happen is again its leftmost branch, so we need to push the right subtree then all of its left children.

See Structure Source:
`structure5/BTPostorderIterator.java`

Here, we visit the left subtree, then the right subtree, and finally the root.

This is the tricky one. First, if there is a left subtree, we need to push down through those left subtrees as far as we can. If any node has no left subtree but has a right subtree, push that instead. Continue to a leaf.

A `next` operation involves popping the top value to be returned. If the thing we just popped is a left child, push the sibling and its left children (or right when there is no left) until we get to a leaf again.

See Structure Source:
`structure5/BTLevelorderIterator.java`

Here, we visit the tree level by level.

This one is actually quite easy. We have a queue instead of a stack.

We start by enqueueing the root, as this is the first thing we want to visit.

When we visit a node, we enqueue its children.

The `BTTraversals` example also demonstrates the use of these iterators.