Additional Exam 1 Review Problems

1. Prove that $2n^2 - 37n + 5 \in O(n^2)$ using limits.

2. Prove that $2n^2 - 37n + 5 \in O(n^2)$ using the definition of O(g(n)).

3. Mark the following as True or False

   _____ $5n^2 \in O(n^3)$ _____ $5n^2 \in \Theta(n^3)$ _____ $5n^2 \in \Omega(n^3)$

   _____ $5n^2 \in O(n^2)$ _____ $5n^2 \in \Theta(n^2)$ _____ $5n^2 \in \Omega(n^2)$

   _____ $5n^2 \in O(n)$ _____ $5n^2 \in \Theta(n)$ _____ $5n^2 \in \Omega(n)$

4. For each function below, circle the function on the right that has the same order of growth as it, or circle none if none grow at the same rate.

a) $\lg(n^4)$ grows at the same rate as: lg n n n lg n n^2 n^3 n^4 2^n none

b) $2^{\lg n}$ grows at the same rate as: lg n n n lg n n^2 n^3 n^4 2^n none

c) $2^{(n+2)}$ grows at the same rate as: lg n n n lg n n^2 n^3 n^4 2^n none

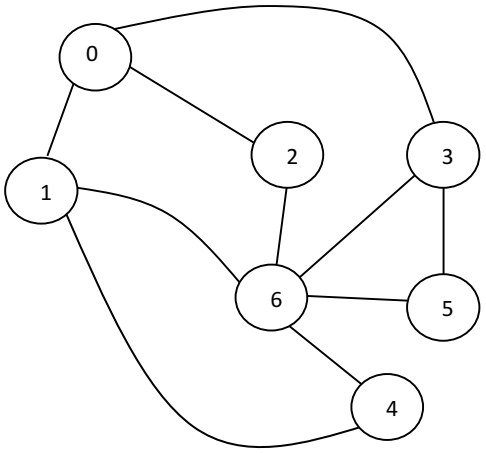
d) $\lg(2^n)$ grows at the same rate as: lg n n n lg n n^2 n^3 n^4 2^n none

5. Prove that n lg n $\in O(n\sqrt{n})$ using limits.

6. (5 Points) Compute a closed form for the following summation. Show your work.

$$\sum_{g=0}^{n-1}\left(n+\sum_{k=1}^{g}5\right)$$

7. (3 Points) Draw the depth-first search tree for the graph G below, starting at vertex 0. When considering "each vertex w in V adjacent to v", consider the adjacent vertices in numerical order from lowest to highest. In other words, if vertex 3 is adjacent to vertices 6, 5, and 0, then consider the adjacent vertices in the order 0, 5, 6. Draw tree edges as solid lines and back edges as dotted lines.



6. Consider an instance of the knapsack problem consisting of n=20 items. If you solve this problem using exhaustive search, how many possible solutions will you need to evaluate? If your computer can evaluate 1 billion possible solutions per second, what is the largest n for which you could solve this problem in one hour?

7. An equilibrium point in an array is an index k such that the sum of all the values to the left of A[k] equals the sum of all the values to the right of A[k]. For example, for array A[0…8] containing the values [3, -1, 4, 9, 2, -7, 8, 4, 6], the index 6 is an equilibrium point because the values to the left of A[6] sum to 10 and the values to the right of A[6] also sum to 10.

   Write an **algorithm in pseudocode** solving this problem. A correct brute force $O(n^2)$ type implementation earns partial credit. A correct and efficient O(n) implementation earns full credit.

   // Returns the index of an equilibrium point in A, if there is one.
   // If there is no equilibrium point, it returns -1
   ALGORITHM EquilibriumPoint(A[0…N-1])