CSIS385 Algorithms
Lab #1 : Counting Operations

Not everything that can be counted counts, and not everything that counts can be counted.

--- Albert Einstein

Names: ____

Learning goals:

- to practice algorithmic thinking
- to be able to model the number of operations performed in nested loops using summations
- to be able to find closed forms for summations
- to be able to express the relationship between the number of vertices and edges in graphs of various types

1) Examine the two implementations below of BubbleSort from lecture.

BubbleSort(A[0…n-1])	ImprovedBubbleSort(A[0…n-1])

for i ← 0 to n - 2
    for j ← 0 to n – 2
        if A[j+1] < A[j]
            swap A[j+1] and A[j]
	for i ← 0 to n – 2
	    for j ← 0 to n – 2 - i
	        if A[j+1] < A[j]
	            swap A[j+1] and A[j]

a) (10 Points) Consider sorting the following five elements using each of these algorithms. Fill in the chart below by showing the contents of the array after **every iteration of the inner for loop.**

33	**55**	**11**	**44**	**22**

33	**55**	**11**	**44**	**22**

b) (4 Points) If you repeated exercise (a) for a 100 element array, how many lines of output would you generate for each version of the algorithm?

BubbleSort: _______________

ImprovedBubbleSort:________________

2) (4 Points) For each sum below, first express it using a summation symbol and then calculate the sum.

   a. 5 + 6 + 7 + 8 + … + 348 =

   b. 1 + 2 + 4 + 8 + 16 + 32 + … + 1024 =

3) (12 Points) Compute closed forms the following sums. In the left column, answers should be integer values. In the right column, answers should be formulas involving n. You may find page 476 of your textbook useful!

$$\sum_{i=3}^{20} 1 \qquad\qquad \sum_{i=3}^{n} 1$$

$$\sum_{i=3}^{20} i \qquad\qquad \sum_{i=3}^{n} i$$

$$\sum_{i=3}^{20} 2^i \qquad\qquad \sum_{i=3}^{n} 2^i$$

4) (10 Points) For the Java code fragment below, complete questions a-d.

```
for ( i = 3; i <= N; i++ )
   for ( k = 0; k < Math.power(2,i); k++ )
      sum = sum + A[i-1];
```

a. For N=5, trace the code and count the number of executions of the basic operation contained inside the inner loop. Write your answer below.

b. Write a general formula involving nested summations that counts the exact number of times the body of the innermost loop executes for any N. Use two summations, one for each loop.

c. Simplify your formula to get a closed form.

d. Now check your formula by substituting in N=5 and see if you get the same as in part a.

5) (10 Points) For this exercise, we will use the standard notation G = (V,E) to describe a graph. When working with graphs, it is often important to be able to bound the number of edges in terms of the number of vertices. Here you get to think about the number of edges in two types of (undirected) graphs. In the chart below, begin by drawing examples of the graph for the specific values of |V| indicated. For each value of |V|, fill in the blank with the number of edges |E| that the graph has. Then come up with a general formula for the exact number of edges in any |V| vertex graph of that type.

	Example Graphs	Formula for Exact Number of Edges |E| (expressed in terms of |V|)
Perfect binary tree of |V| vertices	|V| = 3 |E| = __________	
	|V| = 7 |E| = __________	
	|V| = 15 |E| = __________	
Complete graph of |V| vertices	|V| = 4 |E| = __________	
	|V| = 5 |E| = __________	